

# A New Behavioral Model taking into account Nonlinear Memory Effects and Transient Behaviors in Wideband SSPAs

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**Abstract** — Accurate and non quasi-static behavioral models of SSPAs taking into account nonlinear memory effects become of prime importance for a breakthrough in system level analysis and design of wideband digital communication systems. This paper describes a new method to characterize and integrate memory effects in nonlinear behavioral models of SSPAs allowing to reproduce both transient and steady state behaviors.

## I. INTRODUCTION

Nonlinear dispersive effects (memory) standing in solid state power amplifiers are basically due to non-ideal RF matching and biasing circuits embedding active cells, along with thermal aspects.

One of the main challenge of behavioral modeling is first to integrate an accurate description of high frequency and low frequency memory effects and secondly to enable an accurate prediction of power amplifiers performances in real encountered applications (multitones digitally modulated carriers (CDMA/OFDM)).

Recently, a new behavioral model based on dynamic Volterra formalism has been reported [1][2]. This formalism allows to predict performances of wideband power amplifier, but its validity is limited to system with short term nonlinear memory effects. On the other hand, the model extraction method presented [3], based on two-tone measurements is a bit complex and also the model kernel being extracted from steady state measurement, the prediction of the transient response tends to be poor, although the accuracy of the steady state response is good.

This paper presents an extension of the first order dynamic Volterra model, with a new extraction principle which extends the validity of the model to system with long term nonlinear memory effects and highly improves the accuracy of nonlinear transient responses. The model extraction principle is particularly simple as it will be outlined below.

## II. MODEL PRINCIPLE

Consider  $x(t) = \Re[\tilde{X}(t)e^{j\omega_0 t}]$  and  $y(t) = \Re[\tilde{Y}(t)e^{j\omega_0 t}]$

the input and output signal of a nonlinear system. The aim

of the behavioral modeling is to express the relationship between the input and output envelopes of a nonlinear device with memory, following a convenient mathematical formalism. In this relationship,  $\omega_0$  is a somewhat arbitrary reference frequency usually taken equal to the center frequency of the system operating bandwidth.

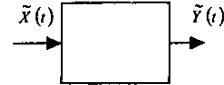


Fig. 1. Nonlinear System with Memory

In general case, the output response at a given time of a device with memory depends on input signal at the same time instant and also input signal at preceding instants in the limit of the memory duration of the system termed  $T_m$ . So, the output response can be written as following :

$$\tilde{Y}(t_n) = f_{NL}(\tilde{X}(t_n), \tilde{X}(t_{n-1}), \dots, \tilde{X}(t_{n-m})) \quad (1)$$

where  $\tilde{X}(t_{n-i})$  represents the input signal envelope at instant  $t_{n-i}$ . The memory duration of the system  $T_m$  is equal to  $t_{n-m} - t_n$ . The function  $f_{NL}(\dots)$  is a whatever nonlinear relation.

If we consider a Taylor expansion of (1) around the time varying DC state  $\tilde{X}(t_{n-i}) = \tilde{X}(t_n) \forall i$ , we obtain the dynamic Volterra expansion reported in [1], which is a rigorous formalism. However, due to the complexity of the Taylor decomposition, in practice, we can apply it only up to the first order. First order truncation of dynamic Volterra expansion permits accurate modeling of highly nonlinear systems [3], but its efficiency tends to be limited only to those systems where the nonlinear memory duration  $T_m$  is sufficiently small compared to the inverse of the bandwidth of the envelope signal  $\tilde{X}(t)$ . Unfortunately, it is shown that most solid state modules such as SSPAs, exhibit an effective nonlinear memory duration much longer than their operating bandwidth inverse especially because of thermal effects [4] and bias circuit modulation effects [5].

In this paper, we propose an extension of first order dynamic Volterra model which handles more effectively long term nonlinear memory effects.

The idea behind is to consider that, in certain conditions, (1) can be effectively approximated by :

$$\tilde{Y}(t_n) = f_0(\tilde{X}(t_n)) + \cdots + f_m(\tilde{X}(t_{n-m})) = \sum_{k=0}^m f_k(\tilde{X}(t_{n-k})) \quad (2)$$

where different functions  $f_k(\cdot)$  are nonlinear relations.

The approximation (2) is particularly true in a system where the memory effect is a secondary effect to a static nonlinear behavior. In such conditions, the individual signal pulses  $\tilde{X}(t_n), \dots, \tilde{X}(t_{n-m})$  propagate nonlinearly in time but tend to sum up linearly. Interesting enough, the long term memory effects due to biasing network and thermal effects fall in this category. For illustration purposes, equation (2) will describe with no error the system depicted in Fig. 2, where a nonlinear amplifier is cascaded with a linear filter.

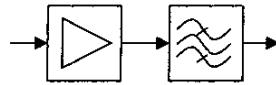


Fig. 2. Nonlinear amplifier cascaded with linear filter

Equation (2) will however handle more complex topologies than this simple illustration. Noting :

$$\frac{f_k(\tilde{X}(t_{n-k}))}{\tilde{X}(t_{n-k})} = h_k(\tilde{X}(t_{n-k})) = h(\tilde{X}(t_{n-k}), t_k) \quad (3)$$

we obtain :

$$\tilde{Y}(t_n) = \sum_{k=0}^m \frac{h(\tilde{X}(t_{n-k}), t_k)}{(t_k - t_{k-1})} \cdot \tilde{X}(t_{n-k}) \cdot (t_k - t_{k-1}) \quad (4)$$

At the limit when  $t_k - t_{k-1} \rightarrow 0$ , we obtain the integral form.

$$\tilde{Y}(t) = \int_0^{T_m} \tilde{h}(\tilde{X}(t-\tau), \tau) \cdot \tilde{X}(t-\tau) \cdot d\tau \quad (5)$$

Considering  $|\tilde{X}(t)|$  and  $\phi_{\tilde{X}(t)}$ , amplitude and time varying phase of  $\tilde{X}(t)$ , we can rewrite :

$$\tilde{Y}(t) = \int_0^{T_m} \tilde{h}(|\tilde{X}(t-\tau)|, \phi_{\tilde{X}(t-\tau)}, \tau) \cdot \tilde{X}(t-\tau) \cdot d\tau \quad (6)$$

Finally, we observe that for a time invariant system, the impulse response  $\tilde{h}(|\tilde{X}(t)|, \phi_{\tilde{X}(t)}, \tau)$  is invariant with the

phase variable  $\phi_{\tilde{X}(t)}$ , so the output equation of the new model becomes :

$$\tilde{Y}(t) = \int_0^{T_m} \tilde{h}(|\tilde{X}(t-\tau)|, \tau) \cdot \tilde{X}(t-\tau) \cdot d\tau \quad (7)$$

Comparison of equation (7) with the first order dynamic Volterra expansion [1],

$$\begin{aligned} y(t) &= \tilde{Y}_{DC}(\tilde{X}(t)) + \int_0^{T_m} \tilde{h}_1(|\tilde{X}(t)|, \tau) \cdot (\tilde{X}(t-\tau) - \tilde{X}(t)) \cdot d\tau \\ &= \int_0^{T_m} \tilde{h}_v(|\tilde{X}(t)|, \tau) \cdot \tilde{X}(t-\tau) \cdot d\tau \end{aligned} \quad (8)$$

shows that this first order Taylor expansion corresponds to the particular case of equation (7) where the nonlinear impulse response  $\tilde{h}(|\tilde{X}(t)|, \tau)$  can be consider almost constant with respect to the signal amplitude, within the memory duration.

$$\tilde{h}(|\tilde{X}(t-\tau)|, \tau) = \tilde{h}(|\tilde{X}(t)|, \tau) \quad \forall 0 \leq \tau \leq T_m$$

Thus (7) is an improvement over the first order Volterra expansion for system with long term nonlinear memory effects.

### III. MODEL EXTRACTION

The impulse response  $\tilde{h}(|\tilde{X}(t)|, \tau)$  can be easily obtained by driving the system with an unit step function  $x(t) = \Re e[X_0 \cdot U(t) \cdot e^{j\omega_C t}]$ , where  $\omega_C$ , the carrier frequency is chosen to be equal to the reference frequency  $\omega_0$  as depicted in Fig. 3.

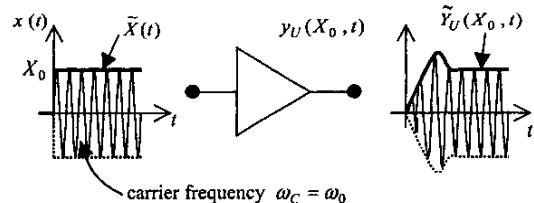


Fig. 3. Impulse Response Extraction Principle

Referring to (7), and applying a step input envelope  $\tilde{X}(t) = X_0 \cdot U(t)$ , the output envelope can be written as following :

$$\tilde{Y}_U(t) = X_0 \int_0^{T_m} \tilde{h}(X_0, \tau) \cdot U(t-\tau) \cdot d\tau \quad (9)$$

Accounting the fact that  $U(t-\tau)=0$  for  $\tau>t$ , and  $U(t-\tau)=1$  for  $\tau<t$ , (9) becomes :

$$\tilde{Y}_U(t) = X_0 \int_0^t \tilde{h}(X_0, \tau) \cdot d\tau \quad (10)$$

The nonlinear impulse response  $\tilde{h}(\dots)$  can thus be readily obtained considering the time derivative of the device response to a step function excitation as expressed in (11). The amplitude  $X_0$  of the step function is swept to cover input power range. The time which the system starts to work in steady state corresponds on memory duration  $T_m$ .

$$\tilde{h}(X_0, \tau) = \frac{1}{X_0} \frac{\partial \tilde{Y}_U(X_0, t)}{\partial t} \quad (11)$$

This type of impulse characterization is nowadays easily carried out using envelope transient analysis [6], available in most of CAD tools. In the last years, we have also observed a development of new measurements tools in time-domain [7][8], well suited to our model extraction. So, the extraction of the proposed model is simple and affordable either through circuit simulation or through calibrated envelope measurements when the device has been fabricated.

#### IV. SLIDING NONLINEAR IMPULSE RESPONSE MODEL

For simplicity, in the preceding section, we supposed that the envelope response  $\tilde{Y}_U(t)$  to the input signal  $x(t) = \Re e \left[ X_0 \cdot U(t) \cdot e^{j\omega_C t} \right]$  was independent of the carrier frequency  $\omega_C$ , and  $\omega_C$  taken to be the center frequency of the system bandwidth. This is not always true in all wide band applications. The dependence of the step response to the input stimulus carrier frequency  $\omega_C$  can be readily taken into account by postulating that the nonlinear impulse response is a function of the instantaneous amplitude and frequency of the incident signal, so that :

$$\tilde{Y}(t) = \int_0^{T_m} \tilde{h} \left( \left| \tilde{X}(t-\tau) \right|, \frac{\partial \phi_{\tilde{X}(t)}}{\partial t} \Big|_{t-\tau}, \tau \right) \cdot \tilde{X}(t-\tau) \cdot d\tau \quad (12)$$

The new function  $\tilde{h}(\dots)$  is called "*sliding nonlinear impulse response*". The extraction procedure is exactly the same as the one presented on Fig. 3, except that the carrier frequency  $\omega_C$  has to be swept through the amplifier bandwidth. Thus, the extraction input stimulus is now :

$$\begin{aligned} x(t) &= \Re e \left[ X_0 \cdot U(t) \cdot e^{j\omega_C t} \right] \\ &= \Re e \left[ \left( X_0 \cdot U(t) \cdot e^{j(\omega_C - \omega_0)t} \right) e^{j\omega_0 t} \right] \end{aligned} \quad (13)$$

Noting  $\Delta\omega_0 = \omega_C - \omega_0 = \partial\phi_{\tilde{X}(t)}/\partial t$  the frequency shift of stimulus carrier with respect to the reference  $\omega_0$ , and referring to (11), we can write

$$\begin{aligned} \tilde{h}(X_0, \Delta\omega_0, t) &= \tilde{h} \left( X_0, \frac{\partial\phi_{\tilde{X}(t)}}{\partial t}, t \right) \\ &= \frac{1}{X_0} \frac{\partial \left( \tilde{Y}_U(X_0, \Delta\omega_0, t) \cdot e^{j\Delta\omega_0 t} \right)}{\partial t} \end{aligned} \quad (14)$$

where  $\tilde{Y}_U(X_0, \Delta\omega_0, t)$  corresponds to output complex envelope around carrier frequency  $\omega_C$ .

#### V. APPLICATION TO A S-BAND AMPLIFIER

The new modeling technique was applied to the circuit design of an S-Band (3 GHz) amplifier. This amplifier was chosen for its recognized memory effects with important impact on performances. Sliding nonlinear impulse response has been extracted by envelope transient simulation.

Comparisons between circuit level simulation results and behavioral simulation results have been performed under several stimuli. The results of the various simulations are presented on the following figures. Fig. 4 presents IM3 curves in case of two-tone stimulus and Fig. 5 and 6, NPR and ACPR characteristics in order to estimate model efficiency with large peak to average ratio excitation.

The new model exhibits a very good accuracy to model long term nonlinear memory. Notably, low frequency variations of IM3 are well reproduced for IM3 up to 15 dBc in a wide band. The IM3 characteristics show a deep resonance at about 10 MHz beat frequency which is a long term memory effect due to bias network. We can note a good versatility of new model as shown by the excellent prediction of NPR and ACPR.

Note that in these simulation, we have consider a NPR stimulus of 50 MHz bandwidth and a QPSK signal of 15MB/s bit rate. The energy of both signal falls well within the frequency band where the memory effects of the amplifier are very important as it can be seen on the IM3 plot (Fig. 4). These figures confirm thus the good nonlinear memory handling capacity of the new model.

Finally, Fig. 7. shows a sample of the time-domain waveform of the output to the QPSK excitation.

Comparison of the new model output with the transient simulation of the circuit gives an appreciation of the good prediction of the amplifier transient response by the new model. A large improvement has been made with respect to the classical memory less model.

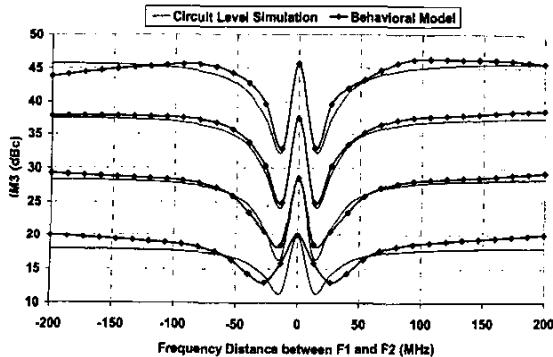


Fig. 4. IM3 (dBc) of New Model / Circuit Simulation in case of Two Tones Signal for Several Input Powers

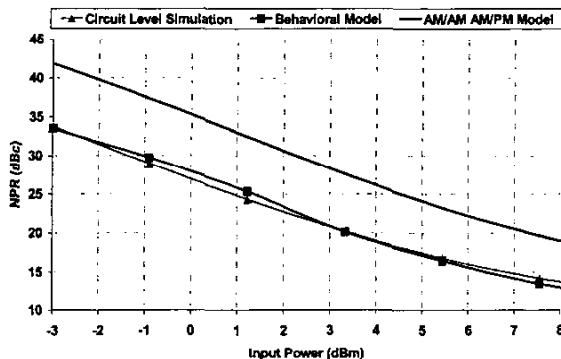


Fig. 5. NPR (dBc) of New Model / Circuit Simulation / AM/AM AM/PM Model

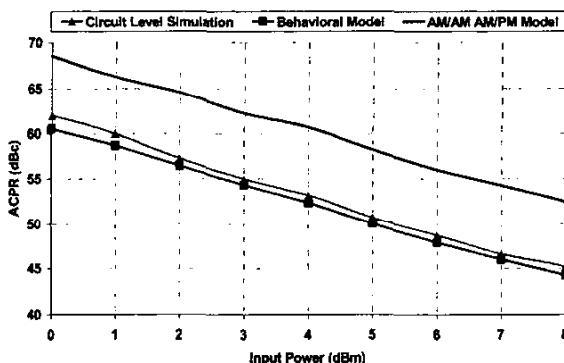


Fig. 6. ACPR with QPSK Input Signal @ Bit Rate = 15MB/s

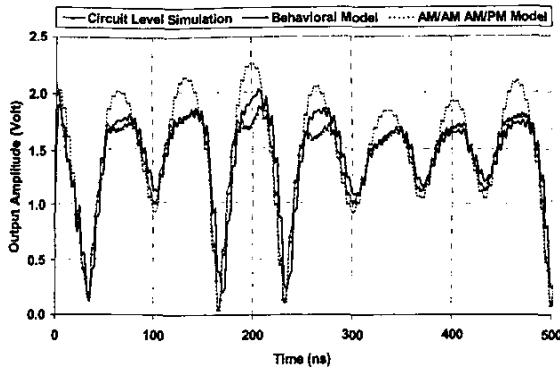


Fig. 7. Time-Domain Waveform Output Amplitude with QPSK Input Signal @ Input Power = 5 dBm, Bit Rate = 15MB/s

## VI. CONCLUSION

An improved modeling technique has been presented. It presents two main interests, first the reproduction of nonlinear memory effects (high and low frequency memory), which allow to use the new model under real applications with good accuracy, and secondly the simplicity of the extraction carried, either by circuit simulation or device time-domain measurements.

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